

## The comparison of models describing the stock return

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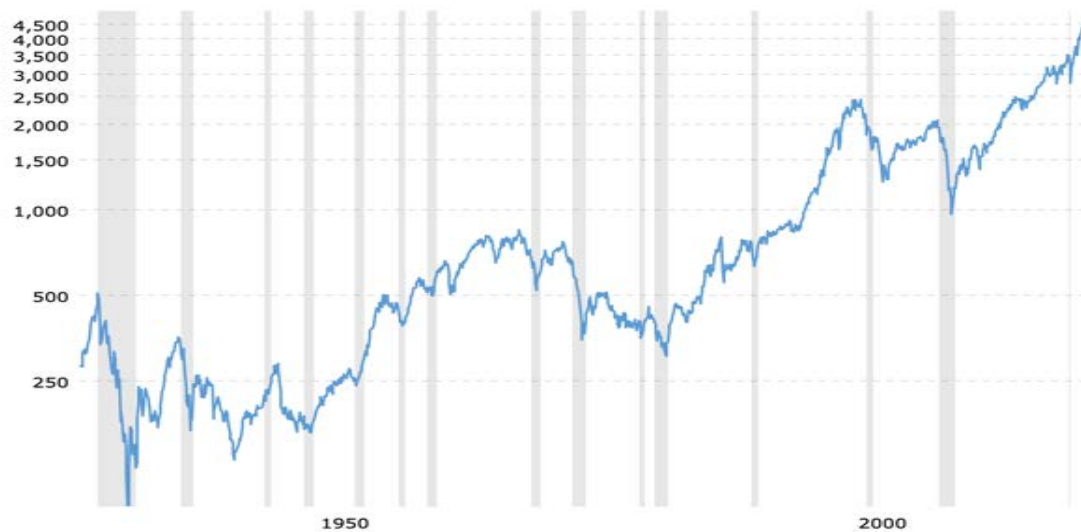
**Abstract:** In this article, I explain several models explaining stock return: capital asset pricing model, three-factor Fama-French model, Carhart four-factor model, and five-factor Fama-French model. About these models, their research methods, statistical analysis, and scientific significance are comprehensively analyzed. According to the development history, the similarities and differences among models are compared. Also, the advantages and disadvantages are evaluated. With the development, models become more accurate in evaluating the stock return. However, each of the models still has its drawbacks, and the regional restriction is one of the most significant problems.

### 1. Introduction

In the stock market, based on the S&P 500 index, from 1982 to 2021, the return of stock always fluctuated. For example, in 2008, the stock index suddenly decrease a lot (see figure 1). To analyze this kind of situation and better predict future indexes, researchers spend a long time building model used to analyses stock returns. They are trying to figure out what factors can influence stock return, which allows them to predict and explain the change of stocks return.

In the development of the factor model, there are capital asset pricing model, three-factor Fama-French model, four-factor model, and five-factor Fama-French model. In the 1960s, the CAPM model using one factor was published to evaluate the investment. Then, in 1993, Fama and French published a three-factor model including the size of firms, book-to-market values, and excess return on the market. The three-factor model can be able to better explain one phenomenon that smaller company's stock can perform better than large companies. After this, in 2012, Mark M. Carhart published a four-factor model by adding the momentum factor. Then, in 2014, Fama and French added two more factors based on the three-factor model, profitability, and investment of firm, which can evaluate the portfolio more comprehensively.

In the rest of the article, I will first talk about Fama-French three-factor model. The research method and its scientific significance of it will be analyzed. Then, I will talk about Fama-French five-factor model. The improvement of this model compared with the three-factor model will be explained. Also, the comprehensive research method and its significance will be talked about. After this, there will be a short analysis of Carhart four-factor model. The difference between it and other models will be compared.



**Figure 1** SP-500 historical chart

## **2. Fama-French three-factor model**

### **2.1. Research Method**

In observation, Eugene Fama and Kenneth French found stocks of small companies and stocks with a high book-to-market ratio tend to have better performance. However, in traditional measurement, CAPM, the prediction does not accurately correspond with the real performance. [1]

To measure the market return more comprehensively, in 1992, Eugene Fama and Kenneth French added two factors, the size of the firm and book to market values, to the CAPM formula to reflect the influence of the small company or the high book to market ratio. Thus, the formula of the Fama-French three-factor model should be that the expected rate of return equals the sum of risk-free rate add market risk premium, the difference between small-cap company's higher returns and a large-cap company's return, and the difference return between the high book to price ratio's stocks and low book to price ratio stocks.[2]

### **2.2. Scientific Significance**

Fama-French three-factor model can calculate the possible return on the stock investment. Also, compared with CAPM, the Fama-French three-factor model can explain more than 90% of the portfolios' returns.

Thus, investors can learn from the model to help them to make the right investment decision. Like Hayes said, because of the volatility and periodic underperformance, the value of the stock may fluctuate. Under this situation, even the loss exists in short term, the value will eventually increase in the long term investment over fifteen years. According to this rule, investors can be more patient with their investments.

Fama-French three-factor model is very significant in real life. In 2010, the article mentioned that, to estimate the cost of developing a new drug, if we use the Fama-French three-factor model, the cost will be higher than using CAPM. This kind of difference reminds the drug company to invest more time to reduce error.[3]

## **3. Fama-French Five-factor model**

### **3.1. Background**

The three factors model cannot include the variation in investment and profitability, which makes the result not comprehensive. And Novy-Marx shows profitability is important in predicting return. [4]

Consider with this, in 2015, Eugene Fama and Kenneth French added two factors, profitability, and investment, into Fama-French three-factor model. Profitability (RMW) is the return difference between a company with high profitability and a company with low profitability. Investment (CMA) is the difference of return between conservative investment and aggressive investment. Like Fama and French wrote in the journal, the five-factor model can describe the average return better than the three-factor model. [5]

### 3.2. Research Method

Fama and French use 5\*5 size-B/M sort, size-OP profitability sort, and size-Investment sort to evaluate the relationships between them.

First, Fama and French examine the relationship between size and B/M, profitability, and investment. They choose 25 portfolios and sort them based on different sizes and B/M. According to the data, we can find the relation between return and size, which is the average return decreases when small stocks are changing to big stocks. This is known as the size effect.

Also, the relationship between B/M and return is also found. When the size does not change, if the B/M increase, the average return will also increase. This is known as the value effect.

Second, using the size-OP profitability sort, the relationship between return and profitability is also examined. When the profitability does not change, as the size increases, the average return will decrease. Also, the high profitability means the return is usually higher than lower profitability's.

**Table 1.** Summary of models

	Time	Advantages	Disadvantages
CAPM	1960s	It has simple calculation and adapts to various portfolios	Cannot show how company size and book to market value influence the stock
Three-factor Fama-French model	1992	Evaluating the stock more comprehensive than CAPM	Global version model is not as accurate as country specific version.
Five-factor Fama-French model	2014	It can explain how investment and profitability influence the stock.	The model does not show how can momentum influence the stock.
Carhart Four factor model	1997	It can show how the momentum influence the stock	The explanation of stock is not comprehensive, and some factors may be ignored.

Third, the relationship between return and investment is also shown in the table. If the size is the same, the low investments lead to higher returns, while the high investments lead to a lower return.

Except for the 5\*5 sorts, Fama and French use 2\*4\*4 sorts to better explain the relationships among size, B/M, OP, and Investment. There are two categories, big size, and small size. And there are three groups in small size: portfolios formed on size, B/M, and operating profitability; portfolios formed on size, B/M, and Investment; and Portfolios formed on size, operating profitability, and investment. Also, there are three same groups with large sizes. Based on the observation of the two tables, we can know how the average return is influenced.

### 3.3. Significance

The Five-factor Fama-French model is widely used in prediction and investing. In 2020, there is an article trying to predict future investment strategies, and their calculation is based on the five-factor Fama-French model.[6] Also, in Australia, the article shows using the five-factor Fama-French model to evaluate the pricing of Australian equity can better explain the anomalies than CAPM.[7] This use of the five-factor Fama-French model proves that it is important in predicting and evaluating investments, and it can be a more comprehensive method.

## **4. Other Model: Carhart Four-Factor Model**

### **4.1. Background**

Except for the three-factor Fama-French model and the five-factor Fama-French model, the Carhart four-factor model which is also known as the Monthly Momentum Factor (MOM) is also important in financial evaluation. This model added momentum factor based on the three-factor Fama-French model. This new factor is slightly better than the three-factor model in evaluating the investment.

### **4.2. Comparing**

As Bello mentioned in the article, in the statistical data, the error variance can be higher when they use the CAPM model. By contrast, using the three-factor Fama-French model or Carhart four-factor model, the error variance will be reduced largely, which means the prediction can be more accurate. Thus, most investors can use the three-factor Fama-French model or Carhart four-factor model to predict the risk of portfolios. [8]

## **5. Discussion**

All these models have different characteristics and drawbacks. For the CAPM model, this one cannot reflect how book-to-market values and company size influence market return. The prediction is not comprehensive and accurate.

For the three-factor Fama-French model, it is better than the CAPM. In the research, the three-factor Fama-French model can explain Swedish stock better than CAPM. [9]

However, the three-factor Fama-French model still faces some regional restrictions. Based on Griffin's article, the country-specific three-factor Fama-French model can provide a better explanation about variation in international stock return than the global version three-factor Fama-French model. This means the three-factor Fama-French model is better to predict a specific country's stock.[10]

About the four-factor model, it is very similar to the three-factor model. In the Swedish stock market, four factors have better performance than CAPM and the three-factor model. The difference between the three-factor model and the four-factor model is small, but the four-factor model is the best one to explain stock in Swedish.

Also, in Teo Lagnilava's research, the momentum factor is significant in explaining AII portfolios. Nevertheless, Carhart four-factor model still has some disadvantages. When the transaction cost is added to this research, the result of 91% stocks is invalid. Also, in the weakest form, because of the relation between momentum and AII portfolio, the four-factor model may lead to wrong results. [11]

About the five-factor Fama-French model, there are some drawbacks. After adding two new factors, the other four factors can explain the stock, so book-to-market values are not necessary. Moreover, the five-factor model also faces regional restrictions. According to James Foye's research, the five-factor Fama-French model cannot explain stocks in UK very well.[12]

## **6. Conclusion**

Referring to the above analysis, all of these models have some drawbacks, and regional restriction is one of the most important ones (see table 1). In both the three-factor Fama-French model and five-factor Fama-French model, they cannot be always accurate to evaluate stocks from different places. This situation indicates the model cannot adapt to various stocks. In future research about the new model, overcoming the regional restriction will be significant. Thus, researchers can spend time finding more related factors that can influence stock. More factors allow the model to be more comprehensive, which helps it to be adaptable to various stocks.

There are some drawbacks of my article which could be better. This article is focused on comparing and analyzing several, and I spend time talking about the model's development history. Thus, the clear connections between models are important to help readers to understand the similarities and

differences among them. Visualizing the development and history of these models may help the reader to comprehensively understand. Currently, some articles use cite space, which is a new application that can visualize and analyze the development pattern. [13] However, in my article, I do not use this kind of mind map to visualize the development history. In this aspect, my article can be improved.

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